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ERRORS DUE TO TERRESTIAL ROTATION RESTITUTION OF LOCALIZATION

Centre National d'Ettudes Spatiales

Translation of Memorandum No. 5/65, Centre National d'Etudes Spatiales, 363, February 1966, 28 pages.
Toulouse, France

(NASA-TT-F-14702) ERRORS DUE TO TERRESTRIAL ROTATION. RESTITUTION OF LOCALIZATION (NASA) Sep. 1972 31 p CSCL

Unclas 46879

N73-1267

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION WASHINGTON, D.C. 20546 SEPTEMBER 1972

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ADVANCED SCHOOL OF AERONAUTICS

Automation Research Center

CNES-363 - February 1966

Memorandum No. 5/65

ERRORS DUE TO TERRESTRIAL ROTATION

RESTITUTION OF LOCALIZATION

February, 1966

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I. ERRORS DUE TO TERRESTRIAL ROTATION

During the time of interrogation T between two positions S_1 and S_2 of the satellite, the balloon is carried along by the rotation of the earth and describes an arc B_1 B_2 whose length depends on the latitude. Any localization directly utilizing the distances d_1 and d_2 (S_1 B_1 and S_2 B_2) will therefore be affected by a systematic error. For an accurate procedure, we must have available, at the instant t_2 , not only d_2 but also a distance d_1^* (d_1 corrected) which will be equal to S_1 B_2 (Figure 1). The error will be as much larger as the bases are closer toward the limits of visibility. However, since the angular velocity of terrestrial rotation is known, it is possible to correct these errors, e.g., by the procedure described below:

Let B_c be the first estimate of the position of the balloon made directly from d_1 and d_2 (the balloon B_c is then localized at the intersection of three spheres: the "sphere" of the balloons, and the spheres S_1 , d_1 and S_2 , d_2). Although inaccurate, this first estimate will give an idea of the latitude ψ of the balloon and consequently of the deviation d_1^* - d_1 . We carry out on B_c the rotation - d_1^* 0 in order to obtain the estimated position d_1^* 0 at the instant d_1^* 1. We then calculate the distance d_1^* 2 and d_1^* 3 (d_1^* 4 and d_1^* 5 (d_1^* 5 and d_1^* 6 and d_1^* 6 and d_1^* 7 and d_1^* 8 first approximation

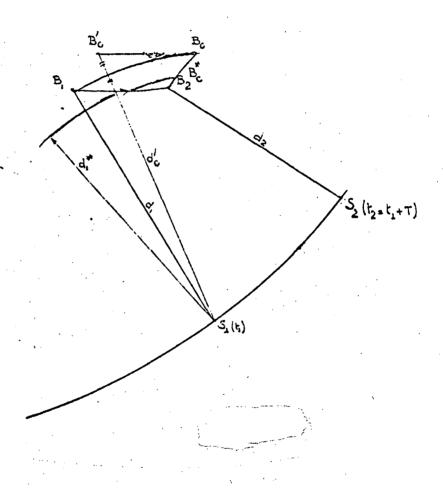


Figure 1

of the error for d_1 (due to the rotation of the earth) is equal to $d_c - d_c^{\dagger}$. We consequently relocalize by utilizing the distances $d_1 = d_1 + d_c - d_c^{\dagger}$, d_2 , and thus obtain a better estimate of B_c which makes it possible to evaluate the correction for d_1 still better, and continue to do so. This iterative process may be automatically arrested when the deviation between two successive localizations will be less than a certain number of meters. We shall see that the process actually converges rapidly.

II. CORRECTION OF ERRORS

2.1 Notations

Correction of errors therefore requires us to be able to effect a first localization B_c of the balloon if we know t_1 , t_2 and d_1 , d_2 . To solve this problem, we use the following trihedra (Figure 2):

- a) The inertial trihedron XYZ where OZ passes through the axis of the poles and where OX is an inertial axis of reference passing through the vernal point.
- b) The plane of the orbit is referenced by its inclination \underline{i} and the inertial longitude Ω of the ascending node. The movement of the satellite in the plane of the orbit is referenced in relation to the trihedron X,

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Y, Z, where OX passes through the ascending node and OZ is carried by the vector $\Omega_{\rm S}$. The formulas of transformation from one to the other system are defined in Table I.

	X,	Υ	£	
X	¢05Ω	Sins	0	
્યુ	-cosisins	$\alpha i \cos \beta$	sini	
X	Sini sin si	- Sini cos s	(os i	

Table 1

Generally, the inertial longitude Ω of the ascending node will be a function of time as well as the radius of the orbit R_S and Ω_S .

c) The rotation of the earth is easily expressed in the absolute trihedron XYZ since we have, between the coordinates of the balloon B_1 at the instant t_1 and those of the same balloon B_2 at the instant t_2 where ω_0 designates the angular velocity of terrestrial rotation:

$$X_{B_2} = X_{B_1} \cos \omega_0 T - Y_{B_0} \sin \omega_0 T$$

$$Y_{B_2} = X_{B_1} \sin \omega_0 T - Y_{B_0} \cos \omega_0 T$$

$$Z_{B_2} = Z_{B_1}$$

Without diminishing the general character of the problem, we can neglect the precession of the orbit and the advance of the perigee in regard to the errors of localization and during the time of visibility in one orbit (1). Under these conditions, we can take the axis OX as the inertial axis OX which simplifies the transformation formulas I $(\Omega = 0)$.

In order to take into account the conditions of visibility, let us assume that the balloon is at the instant t=0 at B_0 at latitude V in the plane XZ (Figure 3) and that the first interrogation is made exactly at the limit of visibility.

⁽¹⁾ We shall return to this problem in paragraph 3 on restitution of the data.

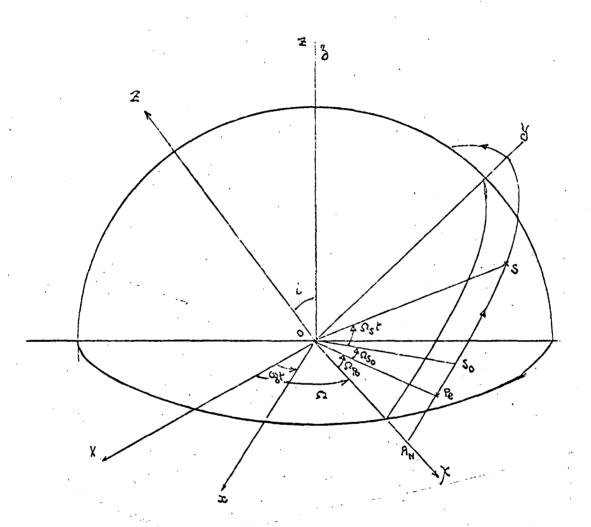


Figure 2

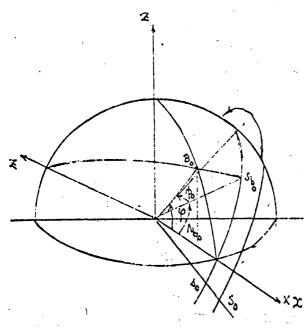


Figure 3

At that instant, the balloon is located at a distance ϕ of from the earth Trace which is such that

- At an elevation \underline{e} , the angle to the center of visibility is \hat{o} such that

and the trace of the satellite at s_0 consequently is such that

In regard to the polar angle of s_{bo} in the plane XY, this is defined by $h_{bo} = arc \frac{1}{3} (sinity)$

The position of the satellite at the start of visibility therefore is

We therefore know the coordinates of the satellite and of the balloon at the instant 0 from the first interrogation. At the instant T, we will have the coordinates of the

balloon by utilizing formulas (1) and those of the satellite by substituting $\[\] \$ for $\[\] \]$ for $\[\] \$ for $\[\] \[\] \$ for $\[\] \$ for

During successive interrogations, we must check, in consideration of the simultaneous displacement of the balloon and of the satellite, that the conditions of visibility remain satisfied. It is sufficient for this to express that the angle (OS_n, OB_n) is less than 0 and consequently that $\cos \alpha = (X_{B_N} X_{S_N} + Y_{B_N} Y_{S_N} + Z_{B_N} Z_{S_N})/RR_S > \cos O$

which makes possible an easy test.

2.2 Localization

The problem is then posed as follows: Knowing the positions of the satellite S_1 and S_2 at the instants of interrogation serving as basis, and knowing the distances d_1 , d_2 between satellite and balloon at these instants, find the position of the balloon on the earth.

If the location will ultimately be made in the system XYZ, we shall begin by referencing the balloon in a new system (\S, \S) defined in Figure 4, where R is the radius of the sphere of the balloons and R_S is the radius of the orbit. The figure lies in the plane of the orbit and the two positions S_1 and S_2 of the satellite are separated by the interval between interrogation T $S_1 S_2 S_2 S_1$. The two spheres S_1 S_1 and S_2 S_2 intersect in a plane perpendicular to S_1 S_2 referenced by the distance mk

$$m_{K} = \xi_{K} = \left(\frac{d_{i}^{z} - d_{z}^{z}}{4R_{s}} \sin M \right)$$
 (3)

The radius r = km of the circle of intersection of the two spheres is equal to $r^2 = d_1^2 - (j_K + R_S \sin p)^2 \qquad (4)$

This circle lies in a plane Γ which intersects the sphere of the balloons along a circle Γ' with its center at H, and with radius r_b such that $r_b^2 = R^2 - s_K^2 \qquad (5)$

The balloon is then localized on this circle Π' by a polar angle Ψ_B $bk = r_b \cos \psi_B = \frac{R_b^2 \cos^2 \mu - r^2 + r_b^2}{2R_a \cos \mu}$

$$\cos \phi_{B} = \frac{r_{o}^{2} - r_{o}^{2} + R_{o}^{2} \cos \phi}{2r_{b}R_{o} \cos \phi}$$
(6)

From the datum of r_b and of ψ_s we deduce the coordinates of the balloon in the system \ref{eq} ?

$$\begin{cases} b = \frac{2(R_{s}^{2} + R^{2}) - (d_{s}^{2} + d_{s}^{2})}{4R_{s} \cos \mu} \\ b = \int_{K} \end{cases}$$

$$\begin{cases} b = \int_{K} \end{cases} \sin \frac{\pi}{2} dx$$

$$(7)$$

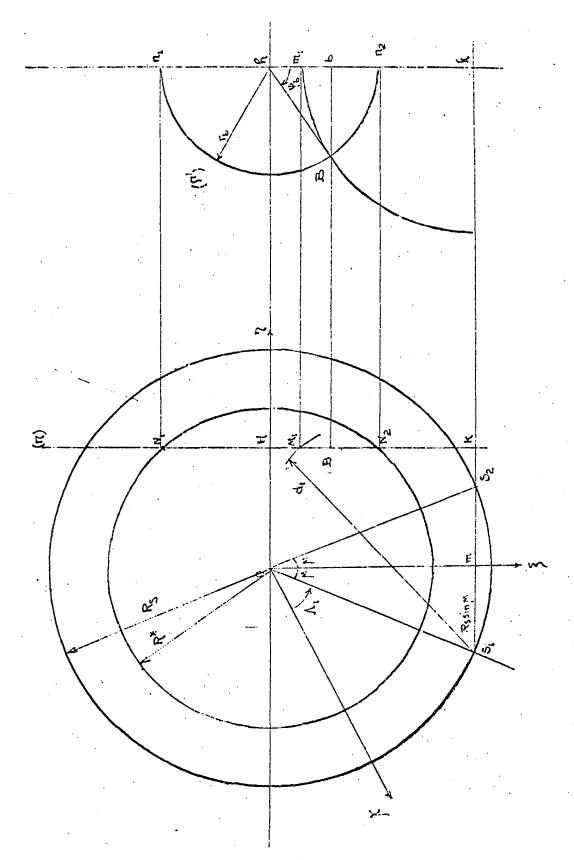


Figure 4

The coordinates of the balloon in the trihedron XYZ are calculated from the transformation formulas

$$X = \xi \cos(\Lambda, +\mu) - i\xi \sin(\Lambda, +\mu)$$

$$Y = \xi \sin(\Lambda, +\mu) + i\xi \cos(\Lambda, +\mu)$$

$$X = \xi \sin(\Lambda, +\mu) + i\xi \cos(\Lambda, +\mu)$$
(8)

With this, the system 2 makes possible localization in the trihedron.

Subsequent calculation for improvement of localization does not raise any difficulties. Its large lines will be found in the annex with the calculation program.

2.3 Results

2.3.1 Influence of the Position of the Bases (cf. program in annex)

It has already been pointed out that, when the bases are at the limit of visibility, the error due to terrestrial rotation may become large. It is thus necessary to test the error and the quality of the proposed iterative process.

For this purpose, we utilized the calculation program Number 1 which will be found in the annex, by effecting the localization on all possible bases.

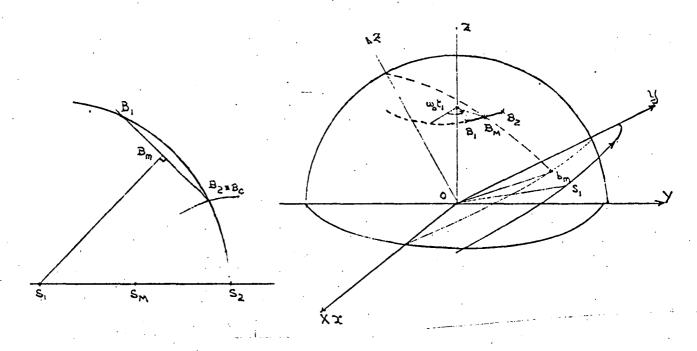
When the interval between interrogation is short in relation to the time of visibility (and we shall see that there is no advantage in increasing this interval too much by reason of the wind), the errors of localization at the limit of visibility become considerable.

The diagrams 1 and 2 are characteristic in this respect: for an orbit of 900 km and inclined at 44° , the error produced has been plotted on the first localization ($B_{c} - B_{v}$) for balloons at various latitudes on the earth and for intervals between interrogation respectively equal to 100 and 200 sec. The localization was made on all successive bases satisfying the conditions of visibility. The deviation between the mean position of the balloon during two successive interrogations in relation to the center of the base utilized has been plotted as abscissa and the error in kilometers on the ordinate. We find that this error may reach $500 \, \mathrm{km}$:

The condition for making the error due to terrestrial rotation zero is the necessity for the sphere with center S_1 and with radius S_1 B_1 = d_1 to also pass through S_2 . We then have d_c = d_1 = S_1 B_2 . This requires that S_1 be located at the intersection of the orbit and of the mean geographic meridian plane of the balloon B_M .

The sequential process of interrogation conditions that this disposition may not correspond to an effective interrogation. The theoretical curve of the error as a function of the angle of the "orbital" meridians of $B_{\!M}$ and $S_{\!M}$ permits a vertical asymptote (zero error in logarithmic coordinates) at the point determined by the above condition. This point can be determined by the

following calculation:



We have:

By
$$Sin(\omega_0t+\frac{\omega_0T}{2})\cos F_1$$
 $Sin(\omega_0t+\frac{\omega_0T}{2})\cos F_1$ $Sin(\omega_0t+\frac{\omega_0T}{2})\cos F_1\cos J$
 XYZ $Sin F_1$ $Sin F_2$

and consequently the polar angle of b_{m}

and consequently, in XYZ, as

$$J \cos J - Z \sin J = X t_g(\omega_0 t + \frac{\omega_0 T}{Z})$$

Straight line OS_1 for Z=0 of polar angle Y/X. By starting with the same initial conditions as in the program given, we have

$$S_{n}$$
: $\Omega_{o}t + (L_{o}-c)$

$$S_{m}$$
: $\Omega_{o}t + (L_{o}-c) + \frac{\Omega_{o}T}{2}$

and consequently the equations:

(1)
$$\operatorname{tg}\left(\Omega_{\delta}t + L_{\bullet} - c\right) = \left(1/\cos J\right) \operatorname{tg}\left(\omega_{\delta}t + \frac{\omega_{\delta}T}{2}\right)$$

(2)
$$V = \left(\Omega_{s}t + L_{o} - C + \frac{\Omega_{o}T}{2}\right) - Arc tg\left(\frac{Sin(\omega_{c}t + \frac{\omega_{o}T}{2})\cos J + tgF, SinJ}{\cos(\omega_{o}t + \frac{\omega_{o}T}{2})}\right)$$

Equation 1 furnishes the t where S_1 passes through the plane of the meridian of $B_{\mbox{\scriptsize M}}$.

Equation 2 makes it possible to calculate the angle corresponding to ob_m with OS_M . A calculation program is given in the annex.

2.3.2 Convergence of the Iterative Process

However, it should be noted that, even in the case where the initial error of localization is very large at the limit of visibility, the convergence of the iterative process is satisfactory. It will obviously require more iterations in the most unfavorable cases for correcting the influence of terrestrial rotation to better than x meters. However, the minor effort of the calculations to be made and their consequent rapidity should not let this be considered as a disadvantage. As an indication, we noted in Table 3 the successive results obtained for an interval between interrogation of 200 sec., and a balloon at latitude 10° when the first interrogation takes place at the limit of visibility. The satisfactory convergence of the process will be noted from examination of the Table since, even in the most unfavorable cases where the initial error may reach 500 km, the error is less than 2.5 km after the third correction and is less than 500 m at the fourth correction and on the order of one meter after the seventh correction.

III.RESTITUTION OF LOCALIZATION

In order to obtain the position of the balloon on the surface of the earth, we need to introduce the complementary system xyz based on the earth. We take oz as passing through the axis of the poles and ox based on the meridian of Greenwich. The geographic coordinates \forall and θ are linked to xyz by

$$x = R \cos \varphi \cos \theta$$

$$y = R \cos \varphi \sin \theta$$

$$\partial = R \sin \varphi$$
(9)

If we assume that, at the time t=0 selected as time base, the meridian of Greenwich forms an angle γ with the inertial axis ox, we will have:

$$X = \infty \cos(y + \omega_0 t) - y \sin(y + \omega_0 t)$$

$$Y = \infty \sin(y + \omega_0 t) + y \cos(y + \omega_0 t)$$

$$Z = 3$$
(10)

In order to take into account the precession of the orbit, the inertial longitude Ω of the ascending node will be a function of time

$$\Omega = \Omega_0 + 0,001624 P^2 \Omega_s \cos i \cdot t$$
 with $P = R/R_s$

If the position of the satellite at a given instant is referenced in relation to the perigee and if, at the time t = 0, the perigee is at $\Omega_{\rm po}$ of OK and the satellite at $\Omega_{\rm so}$ of the perigee, we have $\Lambda_{\rm o}=\Omega_{\rm po}+\Omega_{\rm so}+(\Omega_{\rm s}+\Delta\Omega_{\rm p})$ t with $\Delta\Omega_{\rm p}=0.001624$ ρ^2 $(4-5\sin^2i)\Omega_{\rm s}/2$

We can then locate the satellite in xyz by

$$X_S = R_S \cos \Lambda$$

 $X_S = R_S \sin \Lambda$
 $X_S = 0$

where \mathbf{R}_S and $\boldsymbol{\Omega}_s$ may have values as functions of time in the case of a non-circular orbit.

The processing of the information furnishing localization can therefore be schematized in the manner indicated in Table IV.

IV. CONCLUSION

It appears from this study that it is easily possible to compensate the errors due to terrestrial rotation and subsequently effect an accurate restitution of the localization on the earth.

It should be pointed out that this study was carried out on the assumption that the only errors introduced were those due to the rotation of the earth $^{\omega}$ _o. We shall therefore be compelled later to return to this point. We can actually expect that, when other causes of error intervene e.g., those due to the wind, it will no longer be possible to exactly compensate the errors due to $^{\omega}$ _o because this presupposes that it is possible to exactly determine the latitude of the balloon. Actually, if the errors of localization due to other causes remain below 10 km, which is what is intended,

the influence of $\Omega_{_{\hbox{\scriptsize O}}}$ (missing in source) will result after correction as an error of only a few meters.

This justifies the hypothesis which we shall make hereafter, i.e., the earth a stationary in regard to the evaluation of the other errors. Terrestrial rotation will be reintroduced only subsequently when the problems of restitution arise.

TABLE 3

		base 1	base 2	base 3	base 4	base 5
deviation		-0;39116	-0,19901	-0,00687	0,18527	0,37740
localization	1	277.206	84.235	4.920	166.126	470.958
localization	2	43.690	7.091	4	5.325	65.651
localization	3	6.371	5.96	1	2 56	12.051
localization	4	1.081	51	0	20 .	2.143
localization	5	165	5		3	3 83
localization	Б	25	1		1	68
localization	7	. 4	Ö		. 0	8
localization	3	1			••	2
localization,	9	0				0

CONVERGENCE OF ITERATIVE PROCESS

orbit 900 km

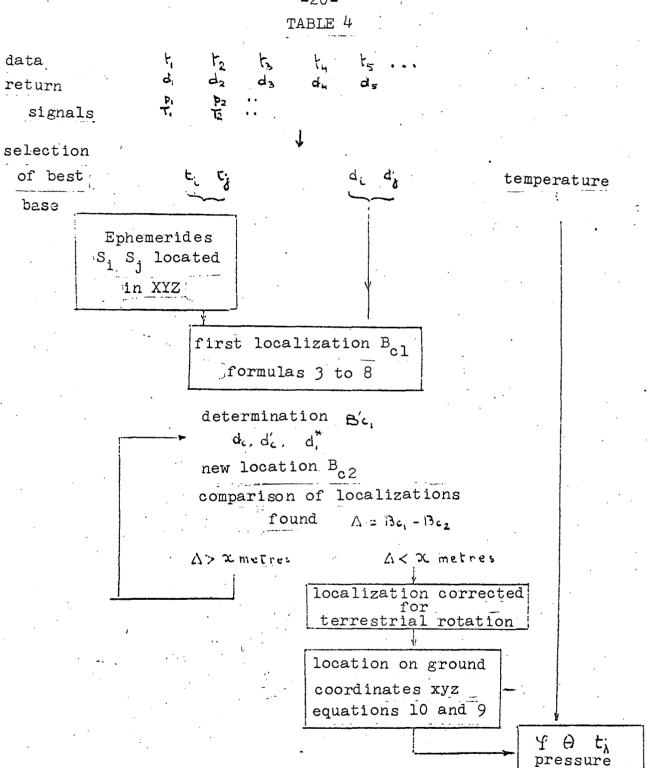
T = 200 seconds

elevation zero

balloon at 10° latitude

first interrogation takes place at the limit of visibility

errors are expressed in meters.



COMPUTATION DIAGRAM

temperature

INFLUENCE OF TERRESTRIAL ROTATION ON LOCALIZATION

$$_{i}$$
 =2 X Y Z H F L

U=R/64(X)

$$P = (398599/R^3)$$

- l Read R J T F₁
- 2 Compute U P
- 3 State Q = 0, 00007268
- 4 Compute $F_2 = ARC SIN ((SIN F_1)(COS J))$
- 5 Go to 30
- 30 Print with 5 DEC TAB R TAB J TAB T TAB F, TAB F, RC
- 31 Compute $C = ARC COS (1/U (COS F_2))$
- 32 Compute $B_0 = COSF_1$
- 33 State $B_1 = 0$
- 34 Compute $B_2 = SIN F_1$
- 35 Compute $L_0 = ARC TG ((SIN F_1)(SIN J)/(COS F_1))$
- 36 Compute $L_1 = L_0 C$
- 37 Go to 39
- 39 Compute $S_0 = U COS L_1$
- 40 Compute $S_1 = U (SIN L_1)(COS J)$
- 41 Compute $S_2 = U (SIN L_1)(SIN J)$
- 42 Compute N = $(B_0 S_0)^2 + (B_1 S_1)^2 + (B_2 S_2)^2$;
- 43 Compute $B_3 = B_0 \cos QT B_1 \sin T$
- 44 Compute $B_{\mu} = B_{0} SIN T + B_{1} COS T$
- 45 Make $B_5 = B_2$

INFLUENCE OF TERRESTRIAL ROTATION ON LOCATION (Continued)

46 Compute
$$L_2 = L_1 + PT$$

48 Compute
$$S_3 = U COS L_2$$

49 Compute
$$S_{\mu} = U (SIN L_2)(COS J)$$

50 Compute
$$S_5 = U (SIN L_2)(SIN J)$$

51 Compute W =
$$B_3 S_3 + B_4 S_4 + B_5 S_5$$

53 Compute
$$G = \sqrt{(B_3 - S_3)^2 + (B_4 - S_4)^2 + (B_5 - S_5)^2}$$

54 Compute A = ARC TG ((
$$B_1 + B_4$$
) COS J + ($B_2 + B_5$) SIN j)/
($B_0 + B_3$))

55 Compute
$$V = (L_1 + L_2)/2-A$$

56 Print with 5 Decimals TAB L
$$_1$$
 TAB L $_2$ TAB A TAB V RC

6 State
$$D = N$$

7 Compute
$$K = (D^2 - G^2)/4U$$
 SIN PT/2

8 Compute
$$B_6 = D^2 (U SIN PT/2 + K)^2$$

9 Compute
$$M = 1 - K^2$$

10 Compute
$$S_6 = ARC COS ((M - B_6 + U^2(COS PT/2)^2)/2U$$

 $(\sqrt{M}) COS PT/2)$

11 Compute
$$X_1 = K SIN (PT/2 + L_1) + (2(1 + U^2) - (D^2 + G^2))$$

(COS (L₁ + PT/2)) 4U COS (PT/2)

12 Compute
$$Y_1 = (K COS (PT/2 + L_1) + (2(1 + U^2) - (D^2 + G^2))$$

$$(SIN (L_1 + PT/2))/4U (COS (PT/2)))(COS J)$$

$$(\sqrt{M}) (SIN S_6) (SIN J)$$

INFLUENCE OF TERRESTRIAL ROTATION ON LOCATION (Continued)

14 Compute E =
$$6400\sqrt{((X_1 - B_3)^2 + (Y_1 - B_4)^2 + (Z_1 - B_5)^2)}$$

- 15 Print with 3 Decimals TAB E RC
- 16 If E<0,01 Go to 23
- 17 Compute $X_2 = X_1 \cos T + Y_1 \sin T$
- 18 Compute $Y_2 = X_1 SIN T + Y_1 COS T$
- 19 Compute $H_1 = \sqrt{((X_2 S_0^2 + (Y_2 S_1)^2 (Z_1 S_2)^2)}$
- 20 Compute $H_2 = \sqrt{((X_1 S_0)^2 + (Y_1 S_1)^2 + (Z_1 S_2)^2)}$
- 21 Make $D = H + H_2 H_1$
- 22 Go to 7
- 23 Make $L_1 = L_2$
- 24 State $B_0 = B_3$ $B_1 = B_4$ $B_2 = B_5$ $S_0 = S_3$ $S_1 = S_4$ $S_2 = S_5$
- 25 Go to 42

END

EXPLANATION OF LETTER SIGNS

 $R:"R_s$ "Radius of orbit

P:" SL" Velocity of satellite rotation

R:" Ω_{o} " Velocity of terrestrial rotation

J: Inclination of orbit

T: Interval between interrogation

 F_1 : Latitude of balloon

F2: Distance from trace (at the initial instant)

C: o "One-half arc of visibility

Bo, B1, B2: Balloon at instant 0

EXPLANATION OF LETTER SIGNS (Continued)

Bo, Bu, Bs: Balloon at instant T

 S_0 , S_1 , S_2 : Satellite at instant 0

 S_2 , S_4 , S_5 : Satellite at instant T

 L_o : " Δ_o " Inertial longitude of balloon B_o

L₁, L₂: Satellite longitudes

G: S_l B_l"

A: Mean position of balloon

V: Deviation between mean positions of balloon and of satellite

K:" ξ" "

B3:" ""

M: nb"

S6: 45"

 X_1 , Y_1 , Z_1 : Balloon calculated as "B"

X2, Y2: Balloon calculated as "B'"

H₇:" d; "

H₂:" d_c"

D: d*"

E: Error of localization

W ← Condition of visibility

INFLUENCE OF TERRESTRIAL ROTATION - CONDITION ZERO ERROR

XAF

_i=2F

 $P = \sqrt{(398599/R^3)}$

U=R/6400

- 1 Read R J
- 2 State T=100
- 3 State $F_{\gamma} = 0.087267$
- 4 Compute A = 1/COS J
- 5 Compute B = SIN J
- 6 Compute $F_2 = ARC SIN ((SIN F_1)/AX$
- 7 Compute U P
- 8 Compute L = ARC TG ((TG F_{γ})B)
- 9 Compute C = ARC COS (1/U COS F₂)
- 10 Read X U
- 11 Compute Y = TG (PX + L C) A TG (X + T/2)
- 12 Make X = X + U
- 13 Compute Z = TG (PX + L C) A TG (X + T/2)
- 14 Make W = YZ
- 15 If W∠0 go to 18
- 16 Make Y = Z
- 17 Go to 12
- 18 Make U = -0, 1U
- 19 If a variant, print with 3 decimals TAB U RC
- 20 If (U) ∠ 0,1 go to 22

INFLUENCE OF TERRESTRIAL ROTATION - CONDITION ZERO ERROR (Continued)

22 Compute
$$V = PX + L - C + PT/2 - ARC TG ((SIN (X + T/2))/A + B TG F1)/(COS (X + T/2))$$

- 23 Print with 5 Decimals TAB F_1 TAB T TAB X TAB V RC
- 24 Make $F_1 = 2F_1$
- 25 If $F_1 > 0,37$ go to 27
- 26 Go to 6
- 27 Make T = 2T
- 28 If T>300 Go to 31
- 29 State $F_1 = 0.087267$
- 30 Go to 6
- 31 End

X: "t" Time

U: Calculation step

R: Radius of orbit

C: One-half arc of visibility

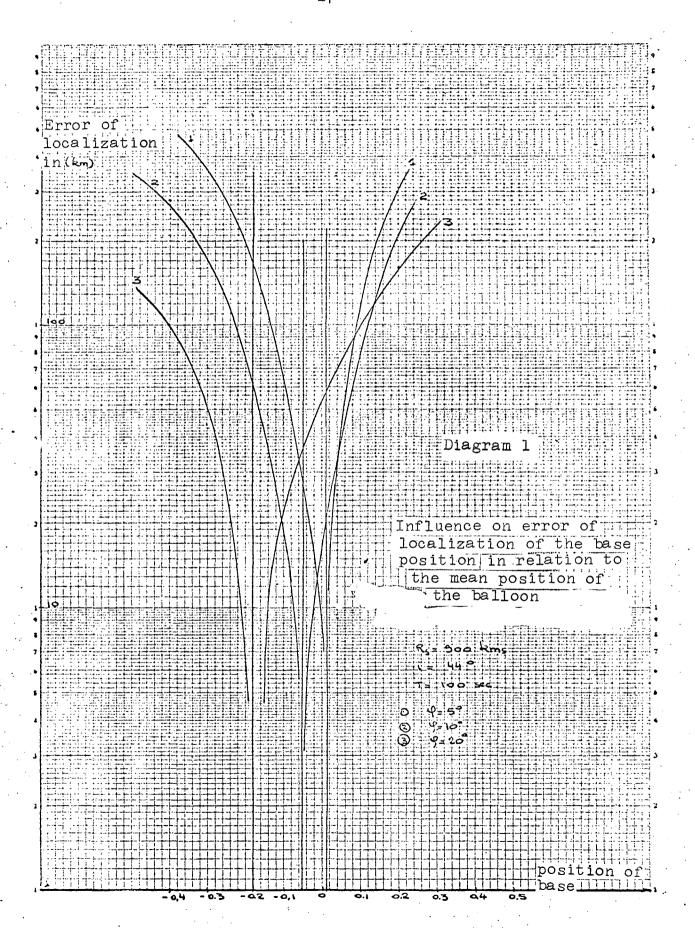
V: "v"

P: "Sha"

Q:"ດ_ດ"

J: Inclination of orbit

L:"Lo"



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